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The πNN Form Factor From QCD Sum Rules

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Abstract

QCD sum rules are used to calculate the q^2 dependence of the πNN coupling $g_{\pi NN}(q^2)$ in the spacelike region $0.5\ \mathrm{GeV}^2 \lesssim q^2 \lesssim 1.5\ \mathrm{GeV}^2$. We study the Borel sum rule for the three point function of one pseudoscalar and two nucleon currents up to order four in the operator product expansion. The Borel transform is performed with respect to the nucleon momenta, whereas the momentum q^2 of the pseudoscalar vertex is kept fixed at spacelike values. The results can be well fitted using a monopole form with a cutoff mass of about $\Lambda_\pi \approx 800$ MeV.

13.75.Gx, 12.38.Lg, 13.75.Cs, 24.85.+p

I. INTRODUCTION

The pion nucleon form factor $g_{\pi NN}(q^2)$ plays a very important, but not less controversial role in the framework of πN and NN dynamics.

In general, meson baryon form factors are used in one boson exchange potentials (OBEP) for the NN force in order to account for the microscopic, i.e. quark and gluon structure of the mesons and baryons at a given vertex, and therefore provide a natural cutoff description for the interaction potential at short distances. Most of the realistic OBEP fit the data with a monopole form for the πNN form factor

$$f_{\pi NN}(q^2) = \frac{g_{\pi NN}(q^2)}{g_{\pi NN}({m_\pi}^2)} = \frac{{\Lambda_\pi}^2 - {m_\pi}^2}{{\Lambda_\pi}^2 - q^2} \tag{1.1}$$

with a large monopole mass $\Lambda_\pi \gtrsim 1.3$GeV [1]. On the other hand almost all other hadronic theories advocate a much softer form factor with $\Lambda_\pi \approx 500 - 950$MeV: Chiral soliton and quark models for the nucleon [2–7], the Goldberger-Treiman discrepancy between $q^2 = 0$ and $q^2 = {m_\pi}^2$ [8], general considerations on the structure of the πNN vertex [9], charge exchange reactions [10], threshold π production [11], deep inelastic lepton nucleon scattering (Sullivan process) [12–14]. More recent studies within the OBEP, which either include the π' resonance [15], use a different form of the scalar exchange potential [16] or consider a correlated $\rho\pi$exchange [17] also indicate a softer cutoff mass ($\Lambda_\pi \approx 700 - 800$MeV).

For these reasons it seems highly desirable to perform a calculation of $g_{\pi NN}(q^2)$, which does not suffer from the ambiguities of the model and parameterization schemes mentioned above and is connected to QCD as closely as possible.

In this context a quenched lattice QCD calculation has recently been carried out rendering a monopole cutoff mass of $\Lambda_\pi = 750$MeV [18].

It is the aim of our work to study this phenomenon within the framework of QCD sum rules [19], which have turned out to be a very successful method for calculating hadronic properties at intermediate energies (for reviews c.f. [20,21]) without employing the computer time consuming lattice gauge calculations.

In order to calculate the pion nucleon coupling constant $g_{\pi NN} = g_{\pi NN}(m_\pi^2)$ itself, i.e. for on shell π, one can consider either: (i) the vacuum three point correlator of two nucleon and one pseudoscalar meson interpolating fields, which are saturated with resonances in the nucleon N and pion π channels on the phenomenological side [20,22–24] or (ii) the 2 point function of 2 nucleon interpolating fields sandwiched between the vacuum and one π state and saturating only with N resonances [20,23,25]. With both methods it was possible to obtain rather reasonable results for $g_{\pi NN}$, although it should be stated that the uncertainties in the Borel analysis are relatively high, especially in method (ii), even if higher order power and α_s corrections are taken into account [25].

If one is interested in the momentum dependence of $g_{\pi NN}(q^2)$ at intermediate q^2 and therefore with off shell π, only method (i), which has been successfully used for the calculation of meson formfactors [26–28], can be applied.

Our paper is organized as follows: In section II we introduce the 3 point function for the $NN\pi$ vertex and saturate with nucleon intermediate states. In section III the operator product expansion (OPE) is carried out. In section IV we perform the Borel analysis. The results are discussed and summarized in section V.

II. THE THREE POINT FUNCTION FOR THE πNN VERTEX

Our starting point is the 3 point function (Fig.1)

$$A(p_1, p_2, q) = \int d^4x_1 d^4x_2 e^{ip_1x_1} e^{-ip_2x_2} \left\langle 0|\mathcal{T}\eta(x_1)J_5^0(0)\bar{\eta}(x_2)|0\right\rangle \tag{2.1}$$

of a pseudoscalar, charge neutral current

$$J_5^0(x) = \bar{q}(x)i\gamma_5\tau^0 q(x) \tag{2.2}$$

and two Ioffe nucleon interpolating fields [29]

$$\eta(x) = \epsilon_{abc}\left[\left(u^a(x)\mathcal{C}\gamma_\mu u^b(x)\right)\gamma_5\gamma^\mu d^c(x)\right] \tag{2.3}$$

Hereby $q = \binom{u}{d}$ denote the spinors for the quarks with mass $m_0 = m_u = m_d$, a, b, c the color indices and $\mathcal{C} = i\gamma_2\gamma_0$ the charge conjugation matrix. The three momenta at the vertex are related by $q = p_1 - p_2$.

Due to restrictions from Lorentz, parity and charge conjugation invariance the 3 point function $A(p_1, p_2, q)$ has the general form

$$A(p_1,p_2,q) = F_1(p_1^2,p_2^2,q^2)\gamma_5 + F_2(p_1^2,p_2^2,q^2)\ \not{q}\gamma_5 + \\ F_3(p_1^2,p_2^2,q^2)\ \not{P}\gamma_5 + F_4(p_1^2,p_2^2,q^2)\sigma_{\mu\nu}\gamma_5 p_1^\mu p_2^\nu \tag{2.4}$$

where $q = p_1 - p_2$, $P = \frac{p_1+p_2}{2}$. The functions F_1,F_2,F_4 are symmetric and F_3 is antisymmetric under the interchange $p_1 \leftrightarrow p_2$.

The matrix element of the pseudoscalar current J_5^0 between on shell nucleon states defines the pseudoscalar nucleon formfactor

$$\left\langle N(p_1)|J_5^0(0)|N(p_2)\right\rangle = g_P(q^2)\bar{u}_N(p_2)i\gamma_5 u_N(p_1) \tag{2.5}$$

where $u_N(p)$ denotes a free nucleon spinor in momentum space.

The pion nucleon coupling constant $g_{\pi NN}$ is defined by the πN interaction:

$$\mathcal{L}_{\pi N} = ig_{\pi NN}\bar{N}i\gamma_5\boldsymbol{\tau}\boldsymbol{\pi}N \tag{2.6}$$

with the form factor $g_{\pi NN}(q^2)$ For on shell nucleons it is equivalent to use the coupling of eq.(2.6) or the chirally invariant vector coupling

$$\mathcal{L}_{int} = \frac{g_{\pi NN}}{2M_N}\partial_\mu\boldsymbol{\pi}\left(\bar{N}\gamma^\mu\gamma_5\boldsymbol{\tau}N\right) \tag{2.7}$$

By means of the chiral Ward identity

$$\partial_\mu\boldsymbol{A}^\mu = m_0\bar{q}(x)i\gamma_5\boldsymbol{\tau}q(x) \tag{2.8}$$

for the axial current $\boldsymbol{A}^\mu(x) = \bar{q}(x)\gamma^\mu\gamma_5\frac{1}{2}\boldsymbol{\tau}q(x)$ as well as the PCAC relation

$$\partial_\mu\boldsymbol{A}^\mu(x) = m_\pi^2 f_\pi\boldsymbol{\pi}(x) \tag{2.9}$$

one can relate the pseudoscalar nucleon form factor $g_P(q^2)$ (eq.2.5) to the pion nucleon form factor $g_{\pi NN}(q^2)$:

$$g_P(q^2) = \left(\frac{m_\pi^2 f_\pi}{m_0}\right) \frac{g_{\pi NN}(q^2)}{-q^2 + m_\pi^2} \tag{2.10}$$

Therefore saturating the eq.(2.1) with nucleon states $|N(p)\rangle$ renders, after continuing to Euclidean momenta ($q^2 \to -q^2$, $p_1^2 \to -p_1^2$, $p_2^2 \to -p_2^2$)

$$[A(p_1,p_2,q)]_N = i\lambda_N^2 \left(\frac{m_\pi^2 f_\pi}{m_0}\right) \frac{g_{\pi NN}(q^2)}{q^2 + m_\pi^2} \frac{[(\not{p}_1 + M_N)\gamma_5(\not{p}_2 + M_N)]}{(p_1^2 + M_N^2)(p_2^2 + M_N^2)} \tag{2.11}$$

where λ_N is the overlap between the Ioffe current $\eta(x)$ and a nucleon state

$$\langle 0|\eta(x)|N(p)\rangle = \lambda_N e^{-ipx} u_N(p) \tag{2.12}$$

Using $p_1^2 = p_2^2 = M_N^2$ the $[\ldots]$ term in eq.(2.11) can be cast into the form

$$(\not{p}_1 + M_N)\gamma_5(\not{p}_2 + M_N) = (-p_1\cdot p_2 + M_N^2)\gamma_5 + i\sigma_{\mu\nu}p_1^\mu p_2^\nu\gamma_5 + M_N \not{q}\gamma_5 \tag{2.13}$$

The contribution of the first higher resonance N^* with mass M_N^* to eq.(2.1) has the form

$$[A(p_1,p_2,q)]_{N^*} = i\lambda_N\lambda_{N^*} \left(\frac{m_\pi^2 f_\pi}{m_0}\right) \frac{g_{\pi NN^*}(q^2)}{q^2 + m_\pi^2} \frac{[(\not{p}_1 + M_N)\gamma_5(\not{p}_2 + M_N^*)]}{(p_1^2 + M_N^2)(p_2^2 + {M_N^*}^2)} \quad + \quad N \leftrightarrow N^* \tag{2.14}$$

III. OPERATOR PRODUCT EXPANSION

As already stated in eq.(2.4) there are essentially 4 Lorentz structures contained in $A(p_1,p_2,q)$. In the following we work with massless quarks, i.e $m_0 = 0$. From a simple dimensional analysis of both sides of the sum rule it is easy to see [20,22,23] that in the sum rule for the functions F_1 and F_4, which both contain an even number of external momenta, only operators of even dimension ($\mathbb{1}$, $\langle G^2\rangle$, $\langle\bar{q}\Gamma q\bar{q}\Gamma q\rangle$, etc.) contribute, whereas the odd dimension operators enter with a mass factor m_0 and therefore vanish for $m_0 \to 0$. On the other hand for the functions F_2 and F_3 containing an odd number of external momenta,

only operators of odd dimension ($\langle \bar{q}q \rangle$, $\langle \bar{q}G \cdot \sigma q \rangle$) contribute. By multiplying both sides of the sum rule with $\not{q}$ and taking the trace over the Dirac matrices, we can single out the odd dimensional structure, i.e. the sum rule for F_2 and F_3. Up to order 4 we only have to account for the diagrams in Fig.2 containing the quark condensate $\langle \bar{q}q \rangle = \langle \bar{u}u \rangle = \langle \bar{d}d \rangle$.

The contribution from the diagrams in Fig.2a renders:

$$[A(p_1,p_2,q)]_{2a} = 16\,\gamma_5\,\langle \bar{q}q \rangle\,\frac{\not{q}}{q^2}\,\frac{1}{2}\,\left[p_1^2 I(p_1^2) + p_2^2 I(p_2^2)\right] \tag{3.1}$$

whereas for the diagrams of Fig.2b one obtains after a rather lengthy calculation:

$$\begin{aligned}
[A(p_1,p_2,q)]_{2b} \;=\; & (\,-16)\,\gamma_5\,\langle \bar{q}q \rangle\,\frac{\not{q}}{4}\,\left[(p_1^2+p_2^2)J(p_1^2,p_2^2,q^2) + (p_1^2-p_2^2)K_+(p_1^2,p_2^2,q^2)\right] \;+ \\
& (\,-16)\,\gamma_5\,\langle \bar{q}q \rangle\,\frac{\not{P}}{2}\,\left[(p_2^2-p_1^2)J(p_1^2,p_2^2,q^2) + (p_1^2-p_2^2)K_-(p_1^2,p_2^2,q^2)\right] \;+ \\
& (\,-16)\,\gamma_5\,\langle \bar{q}q \rangle\,\frac{\not{q}}{4}\,\left[\frac{1}{2}I(p_1^2) + \frac{1}{2}I(p_2^2) + 2I(q^2)\right] \;+ \\
& (\,-16)\,\gamma_5\,\langle \bar{q}q \rangle\,\frac{\not{P}}{2}\,\left[\frac{1}{2}I(p_1^2) - \frac{1}{2}I(p_2^2)\right]
\end{aligned} \tag{3.2}$$

The expressions I,J,K_+ and K_- arise from loop integrations and can be most conveniently represented as double parameter integrals

$$I(p^2) = \int \frac{d^4k}{(2\pi)^2}\frac{1}{(p-k)^2 k^2} = \left(-\frac{i}{16\pi^2}\right)\ln\left(-\frac{p^2}{\mu^2}\right) \tag{3.3}$$

$$\begin{aligned}
J(p_1^2,p_2^2,q^2) &= \int \frac{d^4k}{(2\pi)^2}\frac{1}{k^2(k-p_1)^2(k-p_2)^2} \\
&= \left(-\frac{i}{16\pi^2}\right)(-)\int_0^1 d\rho \int_0^1 d\lambda \frac{1}{p_1^2(1-\lambda)(1-\rho) + p_2^2(1-\lambda)\rho + q^2\lambda\rho(1-\rho)}
\end{aligned} \tag{3.4}$$

and

$$\int \frac{d^4k}{(2\pi)^2}\frac{\not{k}}{k^2(k-p_1)^2(k-p_2)^2} = \left(-\frac{i}{16\pi^2}\right)\left[\frac{\not{q}}{2}K_+(p_1^2,p_2^2,q^2) + \not{P}K_-(p_1^2,p_2^2,q^2)\right] \tag{3.5}$$

with

$$K_\pm(p_1^2,p_2^2,q^2) = \int_0^1 d\rho \int_0^1 d\lambda \frac{\lambda\rho \pm (\lambda-1)}{p_1^2\lambda\rho(1-\lambda) + p_2^2(1-\lambda)(1-\rho) + q^2(1-\lambda)\rho} \tag{3.6}$$

In the UV divergent integral of eq.(3.3) we have applied the standard dimensional renormalization at the renormalization point μ.

IV. SUM RULE AND BOREL ANALYSIS

For the OPE to be valid q^2 has to large, i.e. $q^2 \gg \Lambda_{QCD}^2$. In this case we can neglect the pion mass in the pole term in eq.(3.1), which is consistent with putting $m_0 = 0$.

In refs. [20,22,23] a sum rule for the pion nucleon coupling $g_{\pi NN} = g_{\pi NN}(q^2 = 0)$ has been obtained and analyzed by identifying the residua of the $\frac{1}{q^2}$ pole in eq.(2.11) and in the OPE contribution from Fig.2a (3.1). By introducing

$$\Delta g_{\pi NN}(q^2) = g_{\pi NN}(q^2) - g_{\pi NN}(0) \tag{4.1}$$

and analogous expressions for the higher resonance contributions, we therefore can write down the sum rule:

$$i\lambda_N^2 \left(\frac{m_\pi^2 f_\pi}{m_0}\right) \frac{M_N}{(p_1^2 + M_N^2)(p_2^2 + M_N^2)} \Delta g_{\pi NN}(q^2) \quad + \quad \ldots \quad = \quad \frac{1}{4q^2} \mathrm{Tr}_\gamma \left[\not{q} A(p_1, p_2, q)\right]_{2b} \tag{4.2}$$

Following refs. [26–28] we apply the double Borel transform with respect to the nucleon momenta p_1^2 and p_2^2

$$\mathcal{B}_{12} = \mathcal{B}_1 \cdot \mathcal{B}_2 \tag{4.3}$$

$$\mathcal{B}_i = \lim_{\substack{p_i^2 \to \infty\, ,\, n_i \to \infty \\ M_i^2 = \frac{p_i^2}{n_i}\ \text{fixed}}} \frac{1}{(n_i - 1)!} (p_i^2)^{n_i} \left(-\frac{\partial}{\partial p_i^2}\right)^{n_i} \quad , \quad i = 1, 2 \tag{4.4}$$

and put $M_1^2 = M_2^2 = M^2$. The momentum q^2 of the pion is kept fixed at a spacelike value $q^2 \gg \Lambda_{QCD}^2$, so that both sides of the sum rule depend on M^2 and q^2 now.

After applying the operator $\mathcal{B}_{12}$ on the OPE side the terms which depend only on either p_1^2 or p_2^2 vanish. The Borel transform of the other expressions can be performed by using the relations

$$\begin{aligned}
\mathcal{B}\left\{\frac{1}{p^2 + s}\right\} &= \frac{1}{M^2} e^{-\frac{s}{M^2}} \\
\mathcal{B}\left\{\frac{p^2}{p^2 + s}\right\} &= -\frac{s}{M^2} e^{-\frac{s}{M^2}} \\
\mathcal{B}\left\{(p^2)^k e^{-\alpha p^2}\right\} &= (-)^k \frac{1}{M^2} \delta^{(k)}(\alpha - \tfrac{1}{M^2})
\end{aligned} \tag{4.5}$$

Taking the first resonance with the quantum numbers of the nucleon N^* explicitly into account in addition to the ground state N we obtain the Borel sum rule

$$\begin{aligned}
&\lambda_N^2 \left(\frac{m_\pi^2 f_\pi}{m_0}\right) \frac{M_N}{M^6}\, e^{-2\frac{M_N^2}{M^2}} \, \Big(\Delta g_{\pi NN}(q^2)\Big) \; + \\
&2\lambda_N \lambda_{N^*} \left(\frac{m_\pi^2 f_\pi}{m_0}\right) \frac{M_N + M_N^*}{2M^6}\, e^{-\frac{M_N^2}{M^2}}\, e^{-\frac{M_N^{*\,2}}{M^2}} \, \Big(\Delta g_{\pi NN^*}(q^2)\Big) \; = \\
&g\left(\frac{q^2}{M^2}\right) \frac{\langle \bar{q} q\rangle}{2\pi^2}
\end{aligned} \tag{4.6}$$

where the function g is defined as:

$$g(x) = \int_0^1 d\rho \left[\frac{x}{(1+\rho)^3} \;+\; 2\,\frac{1+\rho^2-4\rho}{(1+\rho)^4}\right] e^{-x\frac{\rho}{1-\rho}} \tag{4.7}$$

In order to eliminate the parameters of the higher resonance contribution as far as possible we can take the derivative $\frac{\partial}{\partial(\frac{1}{M^2})}$ on both sides of eq.(4.7) and substitute the so obtained sum rule back into eq.(4.7). After employing the Gell-Mann–Oakes–Renner relation

$$m_\pi^2 f_\pi^2 = -2 m_0 \langle \bar{q} q\rangle \tag{4.8}$$

we obtain finally

$$\Delta g_{\pi NN}(q^2) = (-)\frac{f_\pi}{M_N}\frac{M_N^6}{4\pi^2\lambda_N^2} B\left(q^2, M^2, M_N^{*\,2}\right) \tag{4.9}$$

with

$$B\left(q^2, M^2, M_N^{*\,2}\right) = \left[g\left(\frac{q^2}{M^2}\right)\frac{M_N^{*\,2} + M_N^2 - 3M^2}{M_N^{*\,2} - M_N^2} + \frac{q^2}{M_N^{*\,2} - M_N^2} g'\left(\frac{q^2}{M^2}\right)\right] \left(\frac{M}{M_N}\right)^6 e^{2\frac{M_N^2}{M^2}} \tag{4.10}$$

Before performing the Borel analysis, let us comment about the region of q^2, where our method can be assumed to work. If q^2 is too low ($q^2 \lesssim \Lambda_{QCD}^2$) the OPE breaks down due to higher order power corrections in $\frac{1}{q^2}$. On the other hand we have used the PCAC relation (2.9) in order to relate $g_{\pi NN}(q^2)$ to the pseudoscalar nucleon form factor $g_P(q^2)$ (eq.(2.10)). This is equivalent to assume pion pole dominance for $g_P(q^2)$, i.e. saturating $\langle N(p_1)|J_5^0(0)|N(p_2)\rangle$ with a pion state and using the coupling $g_{\pi NN}(q^2)$. This assumption

likely breaks down if q^2 is of the same magnitude as the first pionic resonance π', i.e. $q^2 \gtrsim m_{\pi'}^2 \approx 2.2M_N^2$, because then π' contributes in the dispersion relation to a similar extent than π does.

Keeping this in mind we use for the the Borel analysis q^2 values in the interval $0.5M_N^2 \lesssim q^2 \lesssim 1.5M_N^2$, where our approach can be assumed to be reliable. Hereby we follow the spirit of ref. [30] and treat the resonance mass M_N^* as an effective mass, which value is adjusted in order to obtain maximal Borel stability.

From Figs.(3,4,5) we can deduce that for the whole q^2 interval mentioned above an effective resonance mass of ${M_N^*}^2 \approx 6.0M_N^2$ ($M_N^* \approx 2.3$MeV) gives the largest Borel plateau of $0.8M_N^2 \lesssim M^2 \lesssim 1.8M_N^2$.

V. RESULTS AND DISCUSSION

From eqs.(4.9,4.10) we obtain the final expression for the form factor $f_{\pi NN}(q^2)$ defined in eq.(1.1):

$$f_{\pi NN}(q^2) = 1 - \frac{f_\pi}{M_N}\frac{M_N^6}{g_{\pi NN}4\pi^2\lambda_N^2}B(q^2, M^2, {M_N^*}^2) \tag{5.1}$$

Due to the discussion in the last section, we use $M^2 = M_N^2$ and ${M_N^*}^2 = 6.0M_N^2$. For M_N we take the experimental value $M_N = 940$MeV. In order to avoid uncertainties from the sum rule for $g_{\pi NN}$ we will also use the experimental value $g_{\pi NN} = 13.4$ instead of the value from the sum rule of refs. [20,22,23]. The parameter λ_N is not experimentally known but has been determined from various analyses using the nucleon sum rule, the most recent and concerning uncertainties obviously most reliable one coming from ref. [31]:

$${\lambda_N}^2 = 5.5 \cdot 10^{-4}\mathrm{GeV}^6 = 8.07 \cdot 10^{-4}M_N^6 \tag{5.2}$$

As we have discussed in the last section formula (5.2) cannot be applied for low q^2. This means that we have to interpolate from $q^2 \approx 0.5M_N^2$ to $q^2 = 0$, where $f_{\pi NN}(0) = 1$. It is interesting to not that the values for $f_{\pi NN}(q^2)$ which one obtains by this procedure at low q^2

are practically identical to the ones one would get if one applied eq.(5.2) literally to the low q^2 region. This might be purely accidental, but a possible reason could be that higher power corrections in$\frac{1}{q^2}$ blowing up the OPE at $q^2 \to 0$ are canceled or suppressed after applying the double Borel transform $\mathcal{B}_{12}$ (4.4). In order to answer this question one has to examine higher order power corrections to the OPE diagrams in Fig.2, which is rather cumbersome for a 3 point function consisting of four quark lines and will be postponed to a separate analysis.

Anyhow, from Fig.6 we can see that the interpolation of $f_{\pi NN}(q^2)$ from intermediate q^2, where we can apply eq.(5.2), to small q^2 is very smooth. Furthermore in the whole interval $0 < q^2 < 2.0 M_N^2$ the $f_{\pi NN}(q^2)$ of our calculation can be fitted very accurately by a monopole form (1.1) with a cutoff mass of $\Lambda_\pi = 0.85 M_N = 800\text{MeV}$. This result is very close to the one recently obtained from quenched lattice QCD [18], which indicates that both methods give a similar description of the πNN vertex at intermediate q^2.

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REFERENCES

[1] R.Machleidt, K.Holinde and C.Elster, Phys.Rep. **149**, 1 (1987).

[2] T.D.Cohen, Phys.Rev. D **34**, 2187 (1986).

[3] N.Kaiser, U.G.Meissner and W.Weise, Phys.Lett. B **198**, 319 (1987).

[4] S.Theberge et.al., Phys.Rev. D **22**, 2838 (1980).

[5] K.Bräuer et.al., Z.Phys. A **320**, 609 (1985).

[6] P.Alberto et.al., Phys.Lett. B **208**, 75 (1988); Z.Phys. A **336**, 449 (1990).

[7] T.Meissner and K.Goeke, Nucl.Phys. A **524**, 719 (1991).

[8] J.Gasser, M.Sainio and A.Sv̂arc, Nucl.Phys. B **307**, 779 (1988).

[9] D.Schütte and A.Tillemans, Phys.Lett. B **206**, 1 (1988).

[10] H.Esbensen and T.S.H.Lee, Phys.Rev. C **32** 1966 (1985).

[11] T.S.H.Lee, nucl-th-9502005, (1995).

[12] A.W.Thomas, Phys.Lett. B **126**, 97 (1983).

[13] L.Frankfurt, L.Mankiewicz and M.I.Strikman, Z.Phys. A **334**, 343 (1989).

[14] W-Y.P.Hwang, J.Speth and G.E.Brown, Z.Phys. A **339**, 383 (1991).

[15] K.Holinde and A.W.Thomas, Phys.Rev. C **42**, (1990), R1195.

[16] S.Deister et.al., Few Body Syst. **10**, 36 (1991).

[17] G.Janssen et.al., Phys.Rev.Lett. **71**, 1978 (1993).

[18] K.F.Liu et.al., Phys.Rev.Lett. **74**, 2121 (1995).

[19] M.A.Shifman, A.I.Vainshtein and V.I.Zakharov, Nucl.Phys. B **147**, 385; **147**, 448 (1979).

[20] L.J.Reinders, H.Rubinstein and S.Yazaki, Phys.Rep. **127**, 1 (1985).

[21] S.Narison, *QCD Spectral Sum Rules* (World Scientific, Singapore, 1989).

[22] L.J.Reinders, H.Rubinstein and S.Yazaki, Nucl.Phys. B **213**, 109 (1983).

[23] L.J.Reinders, Act.Phys.Pol. **15**, 329 (1984).

[24] S.Narison and N.Paver, Phys.Lett. B **135**, 159 (1984).

[25] H.Shiomi and T.Hatsuda, hep-ph-9504353, (1995).

[26] B.L.Ioffe and A.V.Smilga, Phys.Lett. B **114**, 353 (1982).

[27] V.A.Nesterenko and A.V.Radyushkin, Phys.Lett. B **115** 410 (1982).

[28] B.L.Ioffe and A.V.Smilga, Nuc.Phys. B **216**, 373 (1983).

[29] B.L.Ioffe, Nucl.Phys. B **188**, 317 (1981).

[30] T.Hatsuda et.al., Phys.Rev. C **49**, 452 (1994).

[31] E.M.Henley and J.Pasupathy, Nucl.Phys. A **556**, 467 (1993).

FIGURES

FIG. 1. The three point function $A(p_1, p_2, q)$.

FIG. 2. Diagrams in the OPE

FIG. 3. The Borel curve $B(M^2, q^2, {M_N^*}^2)$ at $q^2 = 0.5{M_N}^2$ for 3 values of the resonance mass ${M_N^*}^2$. Maximum stability is obtained for ${M_N^*}^2 = 6.0{M_N}^2$.

FIG. 4. As in (3) for $q^2 = 1.0{M_N}^2$.

FIG. 5. As in (2) for $q^2 = 1.5{M_N}^2$.

FIG. 6. The $f_{\pi NN}(q^2)$ calculated from eq.(5.2) (interpolated to small q^2) with $M^2 = {M_N}^2$ and ${M_N^*}^2 = 6.0{M_N}^2$ (full line) compared with a monopole form (1.1) with a cutoff mass of $\Lambda_\pi = 0.85M_N = 800$ MeV (dotted line).

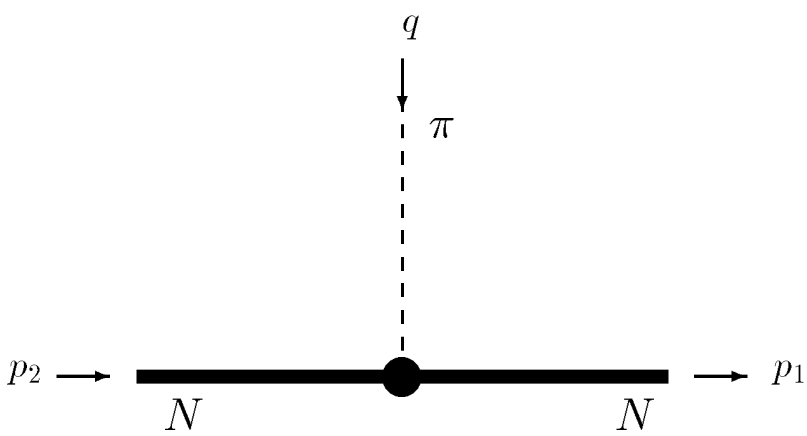


Fig.1

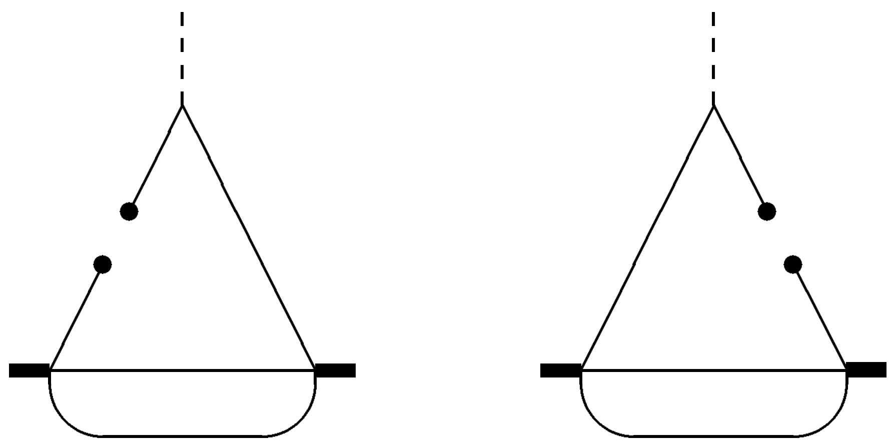


Fig.2a

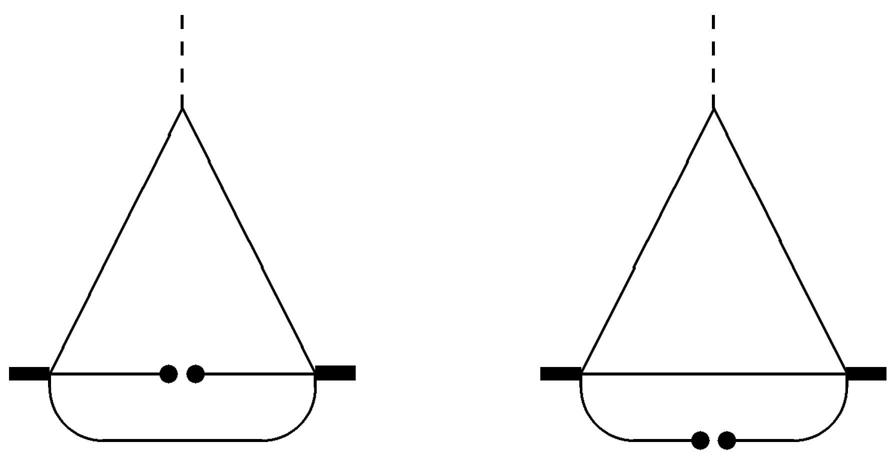
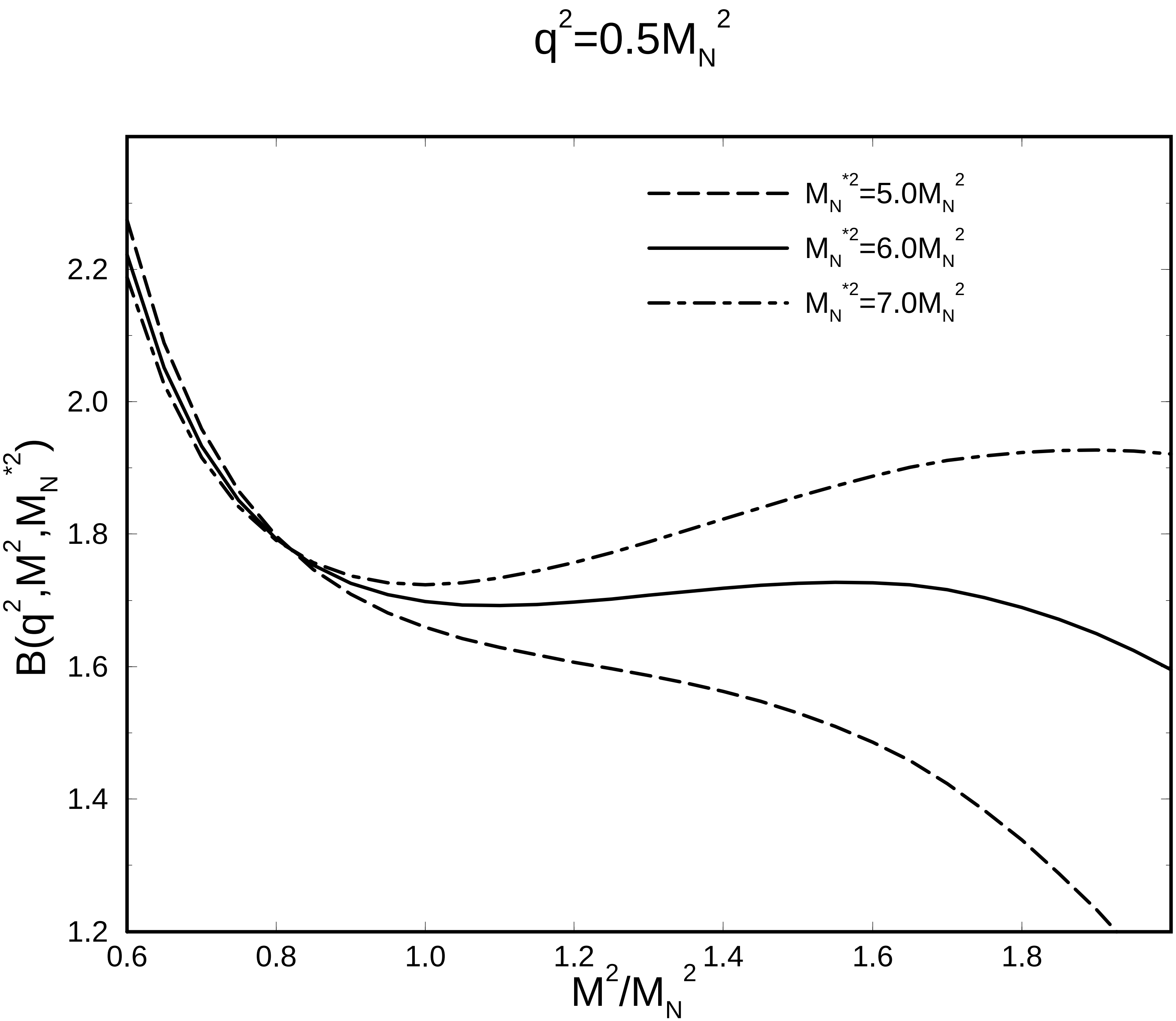
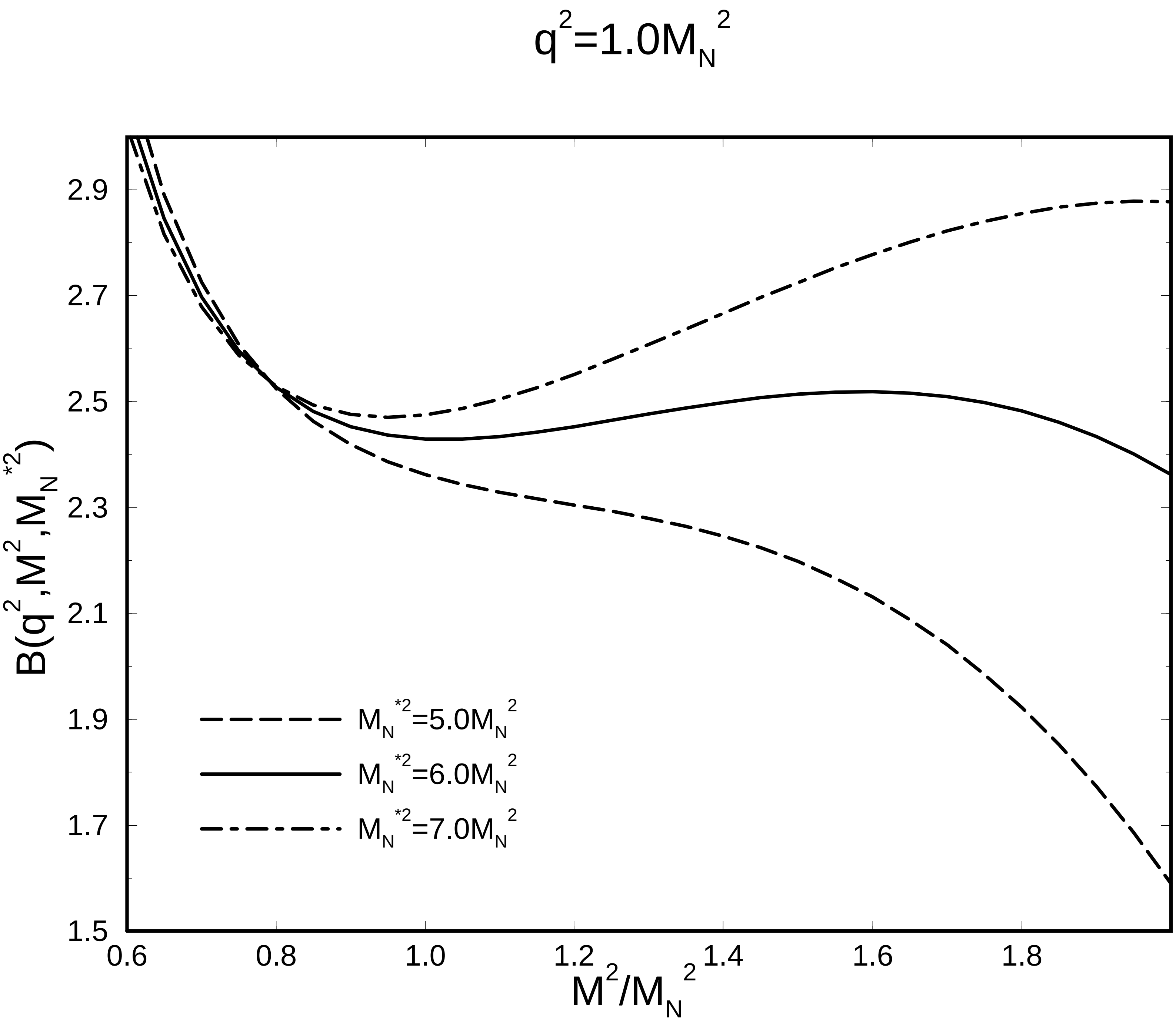


Fig.2b



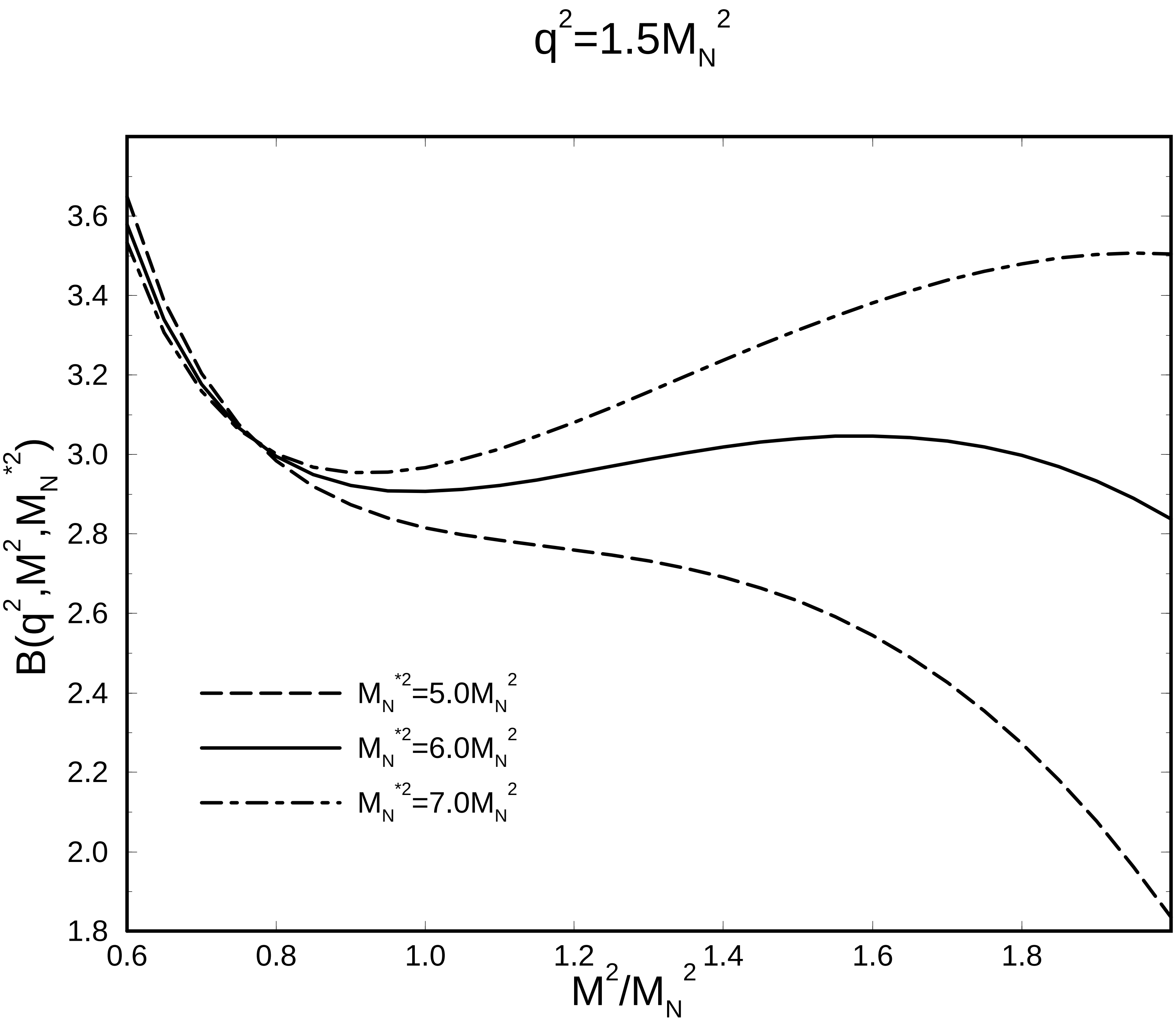
$q^2=0.5M_N^2$
$M_N^{*2}=5.0M_N^2$
$M_N^{*2}=6.0M_N^2$
$M_N^{*2}=7.0M_N^2$
$B(q^2,M^2,M_N^{*2})$
M^2/M_N^2
1.2
1.4
1.6
1.8
2.0
2.2
0.6
0.8
1.0
1.2
1.4
1.6
1.8

Fig.3



q^2=1.0M_N^2
B(q^2,M^2,M_N^*2)
M^2/M_N^2
2.9
2.7
2.5
2.3
2.1
1.9
1.7
1.5
0.6
0.8
1.0
1.2
1.4
1.6
1.8
M_N^*2=5.0M_N^2
M_N^*2=6.0M_N^2
M_N^*2=7.0M_N^2

Fig.4



$q^2=1.5M_N^2$
$B(q^2,M^2,M_N^{*2})$
M^2/M_N^2
$M_N^{*2}=5.0M_N^2$
$M_N^{*2}=6.0M_N^2$
$M_N^{*2}=7.0M_N^2$
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2
3.4
3.6
0.6
0.8
1.0
1.2
1.4
1.6
1.8

Fig.5

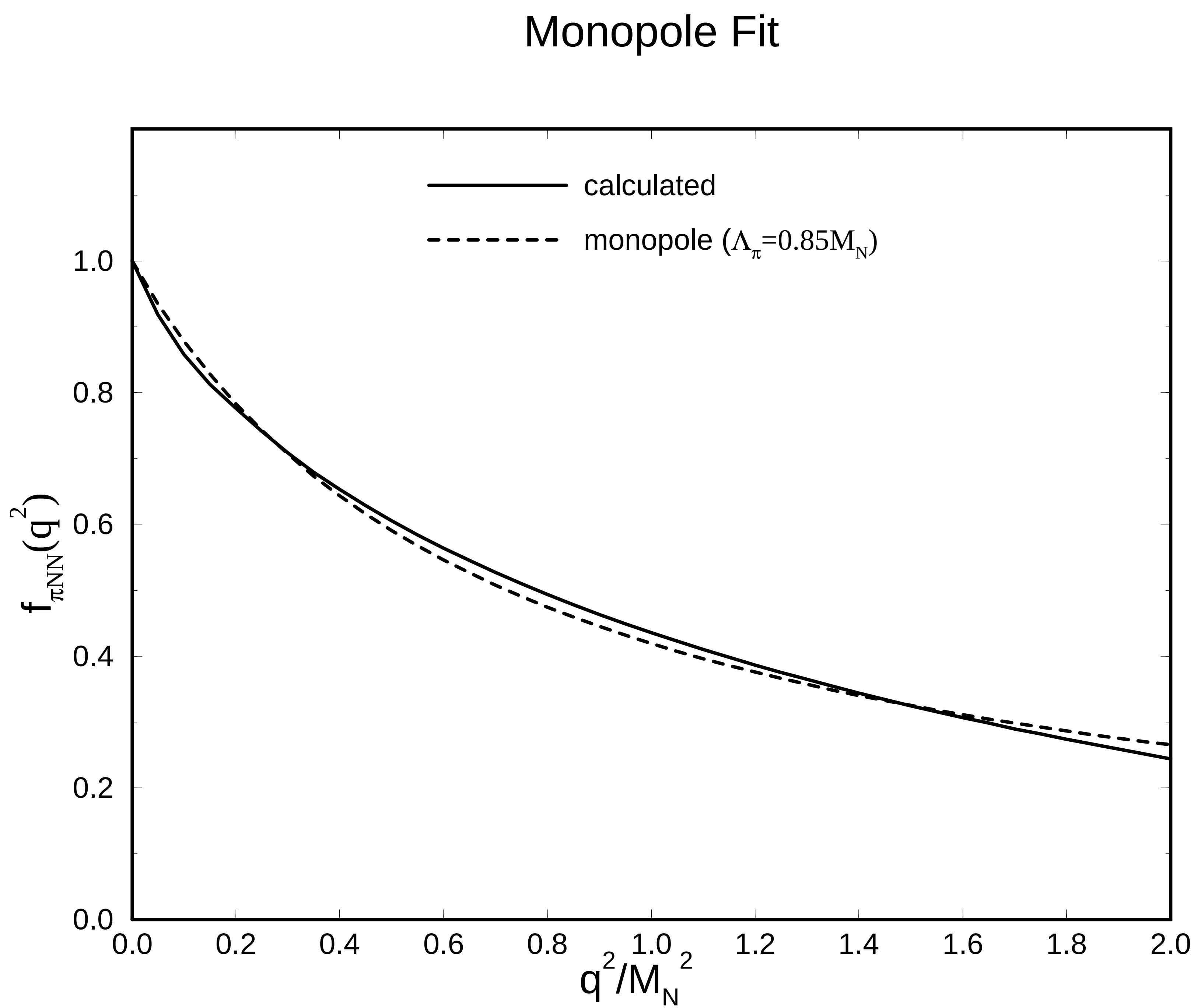

Monopole Fit
calculated
monopole (Λ_π=0.85M_N)
$f_{\pi NN}(q^2)$
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
q^2/M_N^2

Fig.6